SUMMARY OF WHAT TO KNOW!

1.) There will be multiple-choice questions from the posted questions on the class Website – they are split into two pdfs but questions will be drawn from both. As you will have had the chance to see all of the possible questions beforehand, these will be worth 3 pts each on the test. There may also be a few very short answer questions (e.g. yes/no statements about situations, or quick "what's the rotational counterpart to…")

2.) Know how to use the rotational kinematics equations. (I will provide these on the test so you don't have to memorize them.)

3.) Know how to relate a rotating body's angular velocity or angular acceleration to the translational velocity or acceleration of a point on the body. That is, understand how $v = r\omega$ and $a = r\alpha$ work.

4.) Know what the *moment of inertia* tells you, what the definition of the moment of inertia for a group of discrete masses is, and what the moment of inertia for a point mass is. I will provide all other needed moment of inertia relationships on the test.

5.) Know how to use the parallel axis theorem.

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6.) Know the three ways to calculate a torque (by name!) and how to draw r and r_1 .

7.) Be able to determine the DIRECTION of a rotating body's angular velocity vector. Know what the pieces of " $\omega = -(3 \text{ rad/sec})\hat{i}$ " tell you.

8.) Be familiar with the examples we've analyzed a lot with different methods (rotating stick pinned somewhere along its length, rolling ball down ramp, simple Atwood, merry go round).

9.) I will pick, possibly, two of the problems that follow. I would suggest you work in teams to determine how to do each, then get together and talk about each. In any case, you should know the ins-and-outs of each problem.

NOTE: you should focus on problems 1, 2, 3, 4, 5, and 8. The others are interesting and a good brain teaser to see how well you really understand things, but the ones listed above are the ones we will choose from for the exam.

Another note: for each of these, make sure you understand how to <u>derive</u> (meaning start with a governing equation like N2L, conservation of energy, momentum, etc) and <u>fit the equation to the situation</u> (meaning put in the proper variables for a statement that anyone could come in and rearrange to get the final answer).

I.) A massive pulley is used in an Atwood machine. What is known is:

$$m_1, m_2, m_p, R, g, and I_{cm, pully} = \frac{1}{2}m_pR^2$$

You use your finger to keep the pulley from rotating by applying force to the pulley at "R/2" as shown in the sketch (you can assume that force is perpendicular to the radius vector).

a.) Draw a f.b.d. for both masses and the pulley.

b.) How much force must your finger apply to keep the system stationary?

c.) You remove your finger and the system begins to accelerate. What is the magnitude of the acceleration of the masses?

d.) What is the angular acceleration of the pulley?

e.) The mass m_2 drops a distance "h." At the end, what is its velocity?

f.) For #e, what is the angular velocity of the pulley?

g.) For #e, what is the pulley's angular momentum?



2.) A beam of length "L" is pinned at an angle∮ to a wall. Tension in a rope "3L/4" from the pin keeps it in equilibrium. There is a massive lump a distance "5L/6" units up the beam. What is known is:

$$m_{beam}, m_{lump}, L, x, g, \theta, \phi \text{ and } I_{cm, beam} = \frac{1}{12} m_b L^2$$

a.) Draw a f.b.d. for the forces on the beam.

b.) What must the tension in the rope be for equilibrium?

c.) Use the Parallel Axis Theorem to determine "I" <u>of the</u> <u>beam</u> about the pin.

d.) Determine the moment of inertia of the lump, then system, about the pin.

e.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration?

- f.) What is the initial acceleration of the lump?
- g.) The beam's end rotates about the pin. What is angular velocity as it passes through the vertical?

h.) What is the beam's center-of-mass velocity at that point?

i.) What is the beam's angular momentum about the pin at that point?



3.) A thin skinned ball sits on an incline held stationary by your finger (ah, that finger again). What is known is:

$$m_b$$
, R, g, θ , and $I_{cm of ball} = \frac{2}{3}m_bR^2$

a.) Draw a f.b.d. identifying all the forces acting on the ball.

b.) Determine the magnitude of the finger force required to hold the ball in equilibrium.



c.) Use the Parallel Axis Theorem to determine "I" about the point of contact between the ball and the incline.

- d.) What is the acceleration of the ball's center of mass?
- e.) What is the ball's angular acceleration about its center of mass?
- f.) The ball drops a distance "h" from rest. What is the velocity of its center of mass?
- g.) After dropping "h," what's the angular velocity of the ball?
- h.) What is the angular momentum of the ball after dropping "h?"

4.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on a frictionless incline. Your finger keeps everything stationary. Known is:

$$m_1, m_h, m_p, R, g, \theta, and I_{cm of pulley} = \frac{1}{2}m_pR^2$$

a.) Draw a f.b.d. identifying all the forces acting on both masses and the pulley.

 $F_{\text{finger}} = \frac{\mathbf{m}_1}{\mathbf{m}_1}$

b.) Determine the finger's force required to just hold the system in equilibrium.

c.) The finger is removed and the system begins to accelerate. What is its acceleration?

d.) What is the pulley's angular acceleration?

e.) The hanging mass drops a distance "h." What is its velocity at that point?

- f.) What is the angular velocity of the pulley at that point?
- g.) What is the angular momentum of the pulley at that point?

5.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on an frictional tabletop. Your finger perpendicular to the radius vector and a distance R/3 from the axis of rotation maintains motionlessness. Known is:

$$m_1, m_h, m_p, R, g, \mu_k, and I_{cm of pulley} = \frac{1}{2}m_pR^2$$



- a.) Draw a f.b.d. identifying all the forces acting on both masses and the pulley.
- b.) Determine the fore required of the finger to keep the system in equilibrium.
- c.) The finger is removed and the system begins to accelerate. What is the hanging mass's acceleration?
- d.) What is the pulley's angular acceleration?
- e.) The hanging mass drops a distance "h." What is its velocity at that point?
- f.) The hanging mass drops a distance "h." What is the pulley's angular velocity at that point?
- g.) The hanging mass drops a distance "h." What is the pulley's angular momentum at that point?

6.) An arm of length "L" is welded to a pulley of radius R. The system is pinned at the pulley's center of mass. A lump is attached to the pulley/arm's end. The system is initially at stationary. Known:

$$m_{\rm L}, m_{\rm a}, m_{\rm p}, R, g, L, I_{\rm arm's \, c. \, of \, m.} = \frac{1}{12} m_{\rm a} L^2 \text{ and } I_{\rm cm, pully} = \frac{1}{2} m_{\rm p} R^2$$



a.) Draw a f.b.d. for the forces acting on the pulley/arm system. (Hint: Noting that there is only a vertical component acting at the pulley's pin, there are 5 of these forces).

b.) Use the parallel axis theorem to determine the arm's moment of inertia about the pin.

c.) Determine the finger force required to keep the system in equilibrium.

d.) Determine "I" for the lump about the pin. Then determine the total "I" about the pulley's center for ALL of the mass in the system.

e.) With the finger removed, what is the initial angular acceleration of the pulley/arm?

f.) What is the initial magnitude of the acceleration of the lump?

g.) What is the system's angular velocity as the rod passes through the vertical?

h.) At the point alluded to in #g, what is the translational velocity of the rod's center of mass?

i.) At the point alluded to in #g, what is the system's angular momentum about the pin?

7.) A massive hub of radius 2R/3 is glued to a massive pulley of radius R. Your finger acting tangent to the edge of the hub holds the system stationary. Strings are wound around both the hub and pulley with masses attached to each (see sketch). What is known is:

$$m_1, m_2, m_p, R, g$$
, and 4 each disk $I_{cm} = \frac{1}{2}mr^2$,

where "m" in the "l" term is the pulley's mass and "r" its radius.

- m_p F_{finger} m₁
- a.) Draw a f.b.d. for both masses and the pulley/hub assembly.
- b.) How much force must your finger apply to keep the system stationary?
- c.) What is the moment of inertia of the pulley/hub complex.

d.) You remove your finger and the system begins to accelerate. What is the angular acceleration of the pulley/hub?

e.) What is the magnitude of the acceleration of the right hanging mass?

- f.) The right hanging mass freefalls a distance "h." What is its velocity at the end?
- g.) What is the hub's angular velocity for the situation in #e?
- h.) What is the hub's angular momentum for the situation in #e?

8.) A beam of length "L" is pinned at an angle θ a quarter of the way up the beam (i.e., at L/4). Tension in a rope threequarters of the way from the end (i.e, at 3L/4) keeps it stationary. What is known is:

$$m_{beam}$$
, L, g, θ , ϕ and $I_{cm,beam} = \frac{1}{12}m_{beam}L^2$



- a.) Draw a f.b.d. identifying all the forces acting on the beam.
- b.) What must the tension in the rope be for equilibrium?

c.) Use the Parallel Axis Theorem to determine "I" about the pin "L/4" units from the end.

d.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration about the pin?

- e.) What is the initial acceleration of the beam's center of mass?
- f.) The beam rotates downward. What is its angular velocity when it passes through the vertical?
- g.) What is the beam's center-of-mass velocity at that point?
- h.) What is the beam's angular momentum at that point?

9.) A beam of length "L" is pinned at an angle θ with the horizontal. Tension in a rope at an angle ϕ at the end of the beam keeps it in equilibrium. There is a massive lump a distance "2L/3" units from the pin. Known is:

$$m_{beam}$$
, m_{lump} , L, g, θ , ϕ and $I_{cm,beam} = \frac{1}{12}m_{b}L^{2}$

- a.) Draw a f.b.d. for the forces on the beam.
- b.) What must the tension in the rope be for equilibrium?
- c.) Use the Parallel Axis Theorem to determine "I" of the beam about the pin.
- d.) Determine the moment of inertia of the lump about the pin and the total moment of inertia of the system.
- e.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration?
- f.) What is the initial acceleration of the lump?
- g.) The beam and lump rotate downward. What is their angular velocity as they passes through the vertical?
- h.) What is the lump's velocity at that point?
- i.) What is the system's angular momentum at that point?



10.) A hanging mass is attached to a string which is threaded over a massive pulley of radius R and wound around a ball of radius R/2 sitting on an incline. The ball and pulley have the same radius. You additionally know:

$$m_{b}, m_{h}, m_{p}, R, g, \theta, I_{cm of pulley} = \frac{1}{2}m_{p}R^{2}, \text{ and } I_{cm of ball} = \frac{1}{6}m_{b}R^{2}$$

a.) Ignoring the forces acting at the pulley's pin, draw a f.b.d. identifying all the forces acting on both masses and the pulley.

b.) Derive an expression for the hanging mass's upward acceleration.

c.) Determine the angular acceleration of the pulley.

d.) The hanging mass rises from rest a distance "h." What is its velocity magnitude at the top of the rise?

- e.) What is the angular velocity of the pulley at that point?
- f.) What is the angular momentum of the pulley at that point?

 m_p

 m_{h}

 $\mathbf{m}_{\mathbf{b}}$